International Journal of Professional Development, Learners and Learning
2026, 8(2), e2611
ISSN 2754-0618 (Online)
https://www.ijpdll.com/
Research Article
OPEN ACCESS

# An atomic approach to mathematical modeling in sixth-grade students' process of creating real-life situation

Mehmet Bekdemir [1*], Fatih Baş [1]

[1] Department of Mathematics and Science Education, Faculty of Education, Erzincan Binali Yıldırım University, Erzincan, TURKEY
***Corresponding Author:** mbekdemir@erzincan.edu.tr

**Citation:** Bekdemir, M., & Baş, F. (2026). An atomic approach to mathematical modeling in sixth-grade students' process of creating real-life situation. *International Journal of Professional Development, Learners and Learning*, 8(2), Article e2611. https://doi.org/10.30935/ijpdll/18751

**ABSTRACT**

This study aims to examine the mathematical modeling process of sixth-grade students based on the atomic approach in the context of generating real-life situations related to given mathematical models. The study, which used the case study method, was conducted with the participation of 19 sixth-grade students (successful students in mathematics). During the data collection process, instructional experiment practices were employed, and participants were asked to write about real-life situations related to 15 mathematical models (line graphs) provided to them. Video recordings and group worksheets obtained during the process were subjected to content analysis. According to the first result of this study, sixth-grade students, when generating real-life scenarios related to mathematical models based on the atomic approach, use their own experiences (experiential knowledge) as primary sources and the information they gain from sources such as television and social media (environmental knowledge) as secondary sources. However, students show a more active and productive attitude in modeling processes related to their own experiences. According to the second result of the study, mathematical modeling activities based on the atomic approach contribute to students' learning and meaningful use of mathematical concepts they have not yet been formally taught. This result supports the idea that such mathematical modeling activities help students learn abstract mathematical concepts by making them concrete. In light of these results, it is recommended that mathematical modeling activities based on the atomic approach be used not only at the sixth-grade level but also in other middle school grades.

**Keywords:** atomic approach, mathematical modeling, real-life situations, sixth-grade students

Received: 17 Nov. 2025 ◆ Accepted: 14 May 2026

## INTRODUCTION

Mathematical modeling is defined as a cyclical process between the real world and the mathematical world, and it is becoming increasingly important in our daily lives (Common Core State Standards Initiative [CCSSI], 2010). For example, particularly during extraordinary situations such as the COVID-19 pandemic, experts have used various mathematical models to predict the course of the outbreak and present these models to the public. Such situations demonstrate the necessity for individuals to possess mathematical modeling skills, even at a basic level. Consequently, many countries have incorporated mathematical modeling into their curricula, and research in this area has grown.

It is not sufficient for students to merely know the steps of mathematical modeling and its processes. They must also acquire the ability to interpret, refine, and relate models to real life (Carlisle et al., 2025; Zawojewski, 2013). However, conceptual ambiguities regarding mathematical modeling in the literature make it difficult to teach the skills and competencies associated with the modeling process (Blum et al., 2007). Moreover, teachers' limitations in this field and the challenges of implementing modeling activities in classroom settings also negatively affect the process (Gould, 2013). For these reasons, even though mathematical modeling is included in curricula, it is not sufficiently practiced in real classrooms, particularly at lower educational levels (English et al., 2016; Manouchehri, 2017). Research in this area has focused primarily on high school and university levels (Blomhøj & Kjeldsen, 2007; Erbaş et al., 2014; Lingefjärd, 2007).

Nevertheless, some studies have shown that elementary and middle school students can also acquire modeling-related skills at an early age and can make sense of mathematical concepts through mathematical modeling (Carpenter & Romberg, 2004; Manouchehri et al., 2020). For example, English (2012) emphasized that mathematical modeling activities enhance elementary students' model development skills. Other studies also view this developmental stage as ideal for laying the foundation of modeling competencies (English et al., 2016). However, further research is still needed on the modeling competencies of students at these age levels (English, 2021; Leavy & Hourigan, 2018).

On the other hand, most existing research on mathematical modeling has focused on completing the modeling cycle in which a mathematical model is constructed for a given real-life situation and then tested for validity in real life (Kaiser, 2017). However, in classroom practices and research, it is not always necessary to focus on every stage of the modeling cycle (Galbraith et al., 2007). Accordingly, two main approaches are prominent in the implementation of mathematical modeling in classroom settings: the holistic approach and the atomic approach. The holistic approach involves carrying out all stages of the modeling cycle to develop students' modeling skills in an integrated way (Haines et al., 2003). In contrast, the atomic approach focuses on a specific aspect of the mathematical modeling cycle to develop the particular modeling skills of students (Blomhøj & Jensen, 2003). In this study, on the basis of the principle of creating real-life situations suitable for a given mathematical model, the mathematical modeling cycle is examined from an atomic approach perspective, and students' knowledge and skills in this process are investigated.

The purpose of this study is to investigate the mathematical modeling process of sixth-grade students, within the context of creating real-life situations for a given mathematical model, through an atomic approach. Specifically, the study aims to reveal how students establish connections between mathematical models and real-life situations and which sources they utilize during this process.

Within the scope of the study, the following research questions are addressed:

1. What categories of knowledge do students use when connecting mathematical models with real life?
2. How do students establish connections between mathematical models and real life?

## LITERATURE REVIEW

To develop a sound understanding of the concept of mathematical modeling, it is necessary to distinguish between the terms mathematical model and mathematical modeling and to define them conceptually in a transparent manner. In general, mathematical modeling is considered a process, whereas a mathematical model is regarded as the output or product of this process (Anhalt & Cortez, 2016). However, addressing these two concepts separately and in detail within the integrity of the modeling process is highly important for understanding the structure and functioning of modeling.

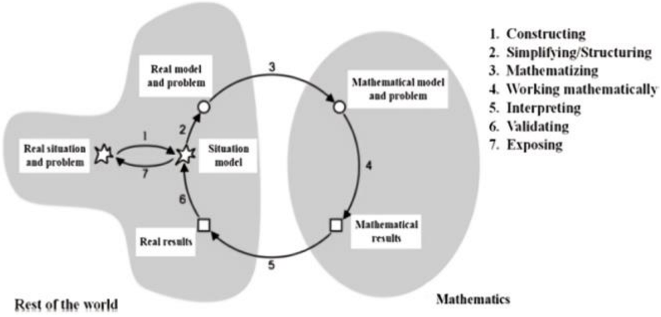


**Figure 1.** The mathematical modeling cycle (Blum & Leiß, 2007)

In the literature, various definitions have been proposed for the concepts of "model" and "mathematical model." A model is defined as a tool that represents not the entirety of real-life situations but specific aspects of them. A mathematical model, in turn, is defined as the expression of the real world through mathematical representations (Greefrath & Vorhölter, 2016). Furthermore, a mathematical model is a tool used to facilitate the comprehension, analysis, and interpretation of a phenomenon or situation in the real world, as well as to make predictions for the future (Lehrer & Schauble, 2007). Mathematical models can take a wide variety of forms, ranging from concrete materials (at the elementary school level) to diagrams, two-way tables, graphs, flowcharts, scaled maps, architectural plans, number lines, and other visual representations, as well as mathematical equations and formulas (Bekdemir et al., 2024; Gould, 2013). Activities designed to generate mathematical models should be derived from students' life experiences to contribute to the development of their mathematical thinking skills. In this way, students acquire the ability to transform real-life situations into mathematical symbolic representations (Lesh & Harel, 2003).

Like the concept of the "mathematical model," there is no consensus in the literature regarding the concept of "mathematical modeling," which has led to the emergence of different definitions. Nevertheless, a standard view in the literature is that mathematical modeling is a cyclical process in which real-life situations or problems are translated into mathematical language and solved within a symbolic system, and the solutions are tested in the real-world context (Blum & Borromeo Ferri, 2009; Lesh & Doerr, 2003). In parallel with the variety of definitions provided, several modeling cycle frameworks have been proposed in the literature (Cirillo et al., 2016). Among these, the modeling cycle developed by Blum and Leiß (2007) (see **Figure 1**) appears to be the most widely referenced framework in mathematics education research (Durandt & Lautenbach, 2020).

The mathematical modeling cycle shown in **Figure 1** consists of six fundamental stages: the real situation, situation model, real model, mathematical model, mathematical results, and real results. Each stage represents a systematic progression toward structuring and solving a real-life situation through mathematical reasoning. Furthermore, the skills and cognitive steps involved in this process are defined in the literature as *mathematical modeling competencies* (MMCs) (Cakmak Gurel & Bekdemir, 2022; Niss et al., 2007). The MMC comprises seven core areas of competence: understanding a given real-life situation; simplifying it within a framework of assumptions and conditions; expressing the constructed real model via mathematical concepts and structures, that is, mathematization; conducting mathematical operations and analyses on the model; interpreting the mathematical outputs in the real context; validating the interpreted results; and finally, presenting the appropriate conclusions. Students often do not follow the steps of the mathematical modeling process in a strictly hierarchical and linear manner (Blum & Leiß, 2007). In contrast, the process typically has a cyclical structure, where students return to earlier steps when necessary and reconstruct the modeling process.

In parallel with the increasing incorporation of mathematical modeling into curricula across all educational levels, research on elementary and middle school students has also increased. It can be classified into two main groups.

The first group of studies aims to design and propose mathematical modeling problems or situations that are applicable at the elementary and middle school levels. For example:

- A mathematical modeling activity for designing a paper airplane considers characteristics such as the flight distance, target landing location, and elapsed time (Lesh & Lehrer, 2003).
- A modeling activity that encourages students to make mathematical inferences from data presented in tables and graphs (English & Watters, 2005).
- “Large number estimation problems” were developed on the basis of Fermi problems, which aim to improve students’ estimation skills (Albarracín & Gorgorió, 2019).
- Modeling activities are inspired by real-life contexts that aim to provide students with different perspectives, such as *expressways* and *Saturday’s candy* (Geiger et al., 2016).

The second group of studies investigates the effects of mathematical modeling activities on elementary and middle school students. For instance:

- English and Doerr (2003) reported that the modeling process enhanced middle school students’ mathematical reasoning, quantitative relation building, and contextual interpretation skills.
- English and Watters (2005) reported that elementary students used their knowledge and experiences in the modeling process, and when this knowledge was insufficient, they focused on the problem context; additionally, modeling activities supported explanations, justifications, and creative discussion skills.
- Mousoulides et al. (2008) noted that sixth- and eighth-grade students who engaged in modeling activities improved their achievement levels, with significant progress observed even among low-achieving students.
- English (2012) reported that in data-driven modeling activities, students represent knowledge in multiple ways (visual, written, and symbolic) and make informal mathematical inferences.
- Wickstrom (2017) reported that mathematical modeling activities improved elementary students’ ability to make choices when confronted with real-life problems, develop alternative solutions, engage in group discussions, and experience success.
- Leavy and Hourigan (2018) reported that modeling activities enhanced the data classification, collaboration, and negotiation skills of children aged five to six years.
- Manouchehri et al. (2020) revealed that group work and appropriate teacher guidance during modeling activities improved fifth-grade students’ mathematical reasoning, their ability to restructure their ideas, and their capacity to construct mathematical expressions.

In conclusion, mathematical modeling lies at the center of daily life (Sokolowski, 2015) and makes multidimensional contributions to students’ mathematical learning. Mathematical modeling activities help students develop a deeper understanding of mathematical concepts, enhance mathematical thinking and problem-solving skills, and foster positive attitudes toward mathematics (Gould, 2013; Suh et al., 2017). In addition, mathematical modeling activities promote students’ mathematical literacy (Steen et al., 2007), enable them to integrate their knowledge and experiences into the problem-solving process (English & Watters, 2005), encourage creative and productive thinking (Lesh & Yoon, 2007), and strengthen their ability to establish connections between mathematical content and real-life contexts (Lehrer & Schauble, 2007). In line with these findings, mathematical modeling has emerged as a practical instructional approach recommended for implementation not only at the secondary and tertiary levels but also across all educational stages beginning from elementary school (Carlson et al., 2016).

## METHOD

This study was designed on the basis of the case study method, which provides opportunities for in-depth data collection and analysis to address the research topic comprehensively (Patton, 2014). The reason for preferring this qualitative research approach is the intention to examine in detail the processes through which sixth-grade students construct real-life situations on the basis of a given mathematical model.

### Participants

The study group consisted of 19 sixth-grade students (11 boys and eight girls) selected from among students with high mathematics and overall academic achievement in a small-scale province located in Eastern Anatolia. The participants were either 12 or 13 years old. Since the research was designed around students' ability to relate mathematical models to real-life situations, it is important to describe the students' individual and environmental characteristics briefly.

Concerning individual characteristics that could influence the research, the students demonstrated high achievement in a mathematics exam administered by the provincial directorate of national education and a private institution specializing in mathematics education. Since the 5th grade, these students have attended classes guided by expert teachers every weekend, for a total of three 40-minute lessons per week. Two of these sessions are devoted to mathematics lessons that emphasize problem solving and are designed with the philosophy of "please make mistakes," which encourages students to express their own ideas and solution strategies freely. The remaining session is allocated to mathematics games.

Notably, the students had limited knowledge of line graphs, which were provided as models in the study. In the Turkish mathematics curriculum, pictograms and object graphs are introduced in the 2nd grade, bar charts in the 4th grade, and double bar charts in the 6th grade. The simple line graphs used in the design of this study are formally introduced only in the 7th grade curriculum. However, in the 6th-grade science and technology course, students are exposed to line graphs in a limited context—specifically, through topics such as "distance-time and speed-time graphs of uniform motion."

The study was conducted in a province with a small population. The city center hosts a university and a world-renowned museum, as well as historical sites such as castle and underground cities. While modern buildings coexist with older structures, the province experiences all four seasons, with harsh and snowy winters when temperatures may fall below zero. The local economy mainly depends on agriculture and animal husbandry, and the province features small forests, waterfalls, and lakes.

### Implementation

This study was designed to evaluate how students connect a line graph, provided as a mathematical model, to real-life situations. The activities were carried out within the framework of the atomic approach to mathematical modeling. Within this framework, the implementation focused on the stages of the mathematical modeling cycle: the mathematical model, the mathematical result, and the real-life situation. To this end, examples of graphs representing various scenarios were selected on the basis of both the literature and real-life context. Two mathematics education experts reviewed these graphs, and 15 graphs were selected for implementation (see **Appendix A**). To ensure reliability, two pilot studies were conducted with these 15 graphs prior to the main study. The pilot studies, which were carried out in two different schools with sixth-grade students, involved three researchers and one observer. On the basis of the pilot results, all 15 graphs were used in the main implementation. The pilot studies also provided valuable insights for both the researchers and the observer, helping to identify critical points for the main study. For example, allowing students to form their groups and present their work using tools such as projectors or smart boards would positively contribute to the process.

Additionally, during whole-class discussions, the researchers repeated the students' questions, responses, or comments aloud to clarify their intended meanings. This approach encouraged the students to focus on the meaning of their responses and facilitated the refinement of their thinking toward more accurate solutions or interpretations. The main implementation was designed and carried out in line with these principles.

The actual implementation consisted of two phases: group work and a whole-class discussion on the basis of the results of the group work. In the first phase, the students formed five groups of four to five members according to their preferences. Worksheets containing the graphs were distributed to the groups, and the students were asked to collaboratively create real-life scenarios corresponding to each of the 15-line graphs. In the second phase, each group selects one of their scenarios to present to the entire class. Other students were encouraged to ask questions or provide comments about the scenario presented, leading to a whole-class discussion. This process lasted for a total of three hours. Two researchers and two mathematics teachers guided the group's work. However, both the researchers and the teachers refrained from immediate interventions when students faced difficulties or raised questions, instead providing minimal support to encourage independence and creativity (Leiß & Wiegand, 2005; Stender & Kaiser, 2015). As presented in the findings section, the limited intervention and support provided by the researchers and teachers enabled the students to express their creativity and develop a wide range of original and diverse ideas.

### Data Collection

The data in this study were collected through video recordings and worksheets prepared by the students. The implementation process, including the activities in the second stage, such as the whole-class discussions, was video recorded. These video recordings were carefully transcribed into written form. In addition, the worksheets on which the groups worked or wrote their scenarios were collected by the researchers for further analysis.

### Data Analysis

The data were analyzed via content analysis through systematic categorization in line with the specific research

**Table 1.** The original and new themes, the content of the themes, and the reasons for the name changes

| Original theme | New theme | Content of the new theme | Reasons for changing theme names |
|---|---|---|---|
| Academic knowledge | Academic knowledge | Theoretical knowledge that is obtained from educational sources. | Used as is. |
| Encyclopedic knowledge | Environmental knowledge | Knowledge that is obtained from sources such as television, social media, etc. | The probability of connecting types of information obtained from books or other reading sources. It has been changed to avoid confusion with academic knowledge. |
| Episodic knowledge | Experiential knowledge | Knowledge obtained from an individual's experiences | The concept of episodic memory is typically associated with memory. Therefore, knowledge that is not obtained from personal experience but from other sources in their environment may also have a memory structure. This category has changed because it is not principally based on memory. |

objectives (Merriam & Grenier, 2019). The analysis process can be summarized as follows:

First, the scenarios written by the students on the worksheets were evaluated in terms of their compliance with the given model, considering both language and meaning. The degree to which these scenarios were realistic and applicable in real-life contexts was subsequently examined. This evaluation process, which was carried out through consensus between two researchers, revealed that a total of 64 scenarios met the specified criteria.

Next, to address the first *sub-research question—"Which sources of knowledge do students use when establishing connections between mathematical models and real life?"*—themes were developed on the basis of the analysis of the students' scenarios. While developing these themes, the thematic framework proposed by Stillman (2000), which classifies the sources of knowledge that students utilize during the mathematical modeling process, was adopted. Stillman (2000) categorized relevant sources of knowledge into three types: academic knowledge (knowledge acquired through academic courses), encyclopedic knowledge (general knowledge about the world), and episodic knowledge (personal and experiential knowledge). These three categories were also adopted in the present study; however, the names of the two categories were modified to reflect their content more accurately. The details of these categories and the rationale behind the name modifications are presented in **Table 1**.

As shown in **Table 1**, in this study, the category of *encyclopedic knowledge* was renamed *environmental knowledge*. The rationale for this change is that the expression "encyclopedic knowledge" may evoke only information acquired through reading and, therefore, carries the risk of being confused with *academic knowledge*. Similarly, the theme of *episodic knowledge* was renamed *experiential knowledge* since the expression "episodic knowledge" may misleadingly suggest knowledge based solely on memory-based individual recollections. This category also encompasses a variety of personal experiences beyond mere memories.

Accordingly, the 64 scenarios written by the students were categorized by two researchers simultaneously and through consensus into three categories: experiential (students' own experiences), environmental (television, social media, etc.), and academic (theoretical knowledge from courses). Detailed information on the relationships between these categories and the students participating is provided in the *Study Group* section. To increase the validity of the classifications, the researchers examined the classroom implementation videos and evaluated the students' explanations (if any) regarding the sources of their scenarios. The frequency and percentage distributions of the scenarios in each category were calculated and tabulated.

The second *sub-research question—"How do students conduct the process of relating mathematical models to real life?"*—was addressed within the framework of the atomic approach, and the data obtained were analyzed accordingly. In this process, instead of considering the entire mathematical modeling cycle proposed by Blum and Leiß (2007), only certain core stages—the *mathematical model, mathematical results, and real* results—were considered, in line with the atomic approach.

Within the atomic approach, students do not follow the modeling cycle in a strictly hierarchical and linear manner. Instead, they tend to engage in a cyclic process, moving forward to the next stage and then returning to the previous stage. For example, a student may start from the *mathematical model* stage, move directly to the *real results* stage, and later return to the *mathematical model* stage.

A similar situation was also observed in this study. After writing a scenario consistent with the given model, students often revise or restructure their scenarios following feedback, questions, or suggestions from their peers during classroom presentations. Throughout this process, they demonstrated core MMCs such as *mathematization, analysis, interpretation, and validation.*

The researchers repeatedly watched the video recordings and carefully reread the transcripts of the students' dialogs for in-depth examination. During this atomic application process, whether the students followed a specific pattern was investigated. When such patterns were identified, the types of MMCs involved and their frequency of occurrence were determined and tabulated. Furthermore, students' discussions and dialogs were analyzed in terms of their use of mathematical concepts. Finally, the relationships between the types of knowledge (defined in the first sub-research question) and these patterns were also analyzed. The data analysis process was

**Table 2.** Frequencies and percentages of the categories of student scenarios

| Category | Frequency (f) | Percentage (%) |
|---|---|---|
| Experiential | 29 | 45 |
| Environmental | 26 | 41 |
| Academic | 9 | 14 |
| **Total** | **64** | **100** |

conducted concurrently and consensually by two researchers, and repeated analyses at different times were carried out to finalize the findings.

## FINDINGS

The findings for each sub-research question are presented in this section in order.

### Findings Regarding the First Sub-Research Question: What Is the Source of the Models Constructed by the Students?

The 64 scenarios written by the participating students on their worksheets were classified according to their experiential, environmental, and academic knowledge. The distributions of these classifications are presented in **Table 2**.

According to the data presented in **Table 2**, the hierarchical ranking of the frequency of categories in the students' scenarios is as follows: experiential, environmental, and academic knowledge. The number of scenarios in the experiential and environmental categories was approximately three times greater than that in the academic knowledge category.

Scenarios related to experiential knowledge reflect students' direct personal experiences or knowledge and ideas derived from their close environment. Examples of such scenarios written by students include "*The number of days required to construct a building,*" "*The change in temperature from one month to another,*" "*The decrease in the integrity of water flowing from a waterfall,*" "*The change in the price of a product depending on the season,*" and "*The time required for human growth and development.*"

Scenarios related to environmental knowledge reflect the knowledge and ideas that students acquire from sources such as television, the internet, and social media. Examples include "*The increase in film ratings,*" "*The increase in the human population in today's world,*" "*The advancement of technology over the years,*" "*The national economy,*" and "*The increase in exports from 2007--2008.*"

Scenarios related to academic knowledge are based on theoretical information acquired in school lessons. Examples provided by the students include "*Real-life situations modeled by cubic functions,*" "*A vehicle moving at constant speed,*" "*The reduction of a person's weight from the north pole to the equator,*" "*The rise and fall of an empire,*" and "*Sound frequency.*"

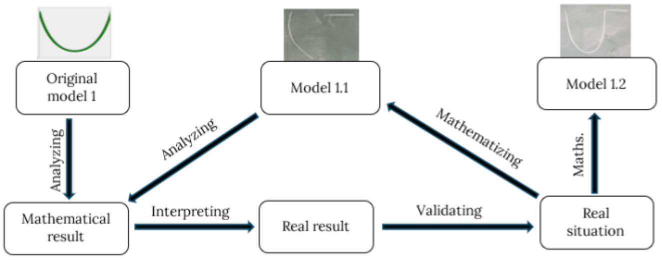


**Figure 2.** The visualized modeling cycle (Source: Authors' own elaboration based on the results of the present study)

### Findings Regarding the Second Sub-Research Question: How Do Students Construct Models of Real-Life Situations?

To answer this sub-research question, video recordings and transcripts were analyzed on the basis of the atomic approach of the mathematical modeling cycle. In the analysis process, the stages of "*mathematical model,*" "*mathematical results,*" and "*real results*" were considered, together with the competencies of mathematization, analysis, interpretation, and validation. As a result of this analysis, three main mathematical modeling patterns were identified. In this section, one example from each pattern is presented, and corresponding cycle structure is explained.

#### *The first pattern*

The first pattern refers to situations in which the real-life scenario constructed for the given mathematical model (the graph) does not sufficiently correspond to the presented graph. In such cases, other students propose alternative graph models that better fit the suggested scenario(s) (see **Figure 2**).

As presented in **Figure 2**, the students generated original alternative models for the given mathematical model. For example, for a mathematical model provided in the form of a graph, one group of students constructed the scenario of "a product's price change according to the season." However, another group of students objected to this scenario, stating that "the price of the product decreased at the end of the season, but the new season price started again at the highest level rather than at the discounted level." To represent this objective, they proposed an alternative model using a piecewise function (model 1.1). A third group of students, assuming that both scenarios could be valid, developed another model (model 1.2) by integrating the two scenarios on the basis of a piecewise function.

The dialogs among the students regarding these alternative graph models were as follows:

> S1 (original model): "... At first, prices are high, then at the end of the season there is a discount, so prices decrease, and afterwards, once the end-of-season discount is over, prices rise again ..."
>
> S2 (model 1.1): "... Why do the new season's products start at the same price as the last season's discounted

products? Logically, the new season's price should be higher. It should start at a different level …"

S3 (model 1.2): "… If the graph did not break, it would rise again in a U-shape, but since it already rose to a certain level, shouldn't it continue steadily after that?"

When students' discussions in this pattern were examined according to the steps and competencies of the mathematical modeling cycle, a total of 36 behaviors were identified. Among these, 25 behaviors were related to the transition from mathematical thinking to real-life contexts, including 18 instances of interpretation and seven instances of validation. Additionally, during the discussion, two alternative models were proposed. In this process, 11 behaviors were categorized under the competencies of mathematizing and analyzing. However, these competencies did not appear in a strictly sequential or cyclical structure, and in fact, some competencies were not observed at all.

Examples of student statements illustrating each competency are presented below:

S14: "… come like this and continue like that (model 1.1). Because it does not keep the same price …" (mathematizing)

S20: "… For example, as it decreases like this, the rate of increase is the same …" (analyzing)

S10: "… S/he does not sell the product at the same price (10 liras) that s/he bought it for. S/he adds some profit again …" (interpreting)

S4: "… Prices are not at the highest level anyway; they drop a little at the end of the season and then fall a larger amount after that. It does not show a direct fall, neither an initial decrease nor a subsequent fall …" (validating)

In terms of mathematical concepts, the students' discussions revealed that these concepts were used and explored in depth. For example, notions such as *decrease, increase, constancy, mathematical interpretation of the graph concerning the x- and/or y-axis*, and *the graph not touching the x- and/or y-axis* were actively employed. Furthermore, the students indicated the maximum and minimum points on the graphs, stating that the highest and lowest values (prices) were obtained at these respective points.

The steps and competencies displayed during this modeling cycle did not follow a sequential or cyclical structure. As illustrated in **Figure 2**, students' transitions between the "mathematical results," "real results," and "real situation" stages of the mathematical modeling cycle were not observed to be strictly ordered or cyclic.

Two noteworthy points emerge from this analysis. First, even though concepts such as piecewise functions and maximum–minimum points are generally beyond students' cognitive levels, they effectively utilize these concepts to express their ideas. Second, when the researcher asked, "*Can this real-life scenario related to the model be accepted as correct?*" one student responded, "*This graph can be read in this way, but it would not be convincing*," demonstrating the ability to question and distinguish a mathematically correct situation within the context of real life.

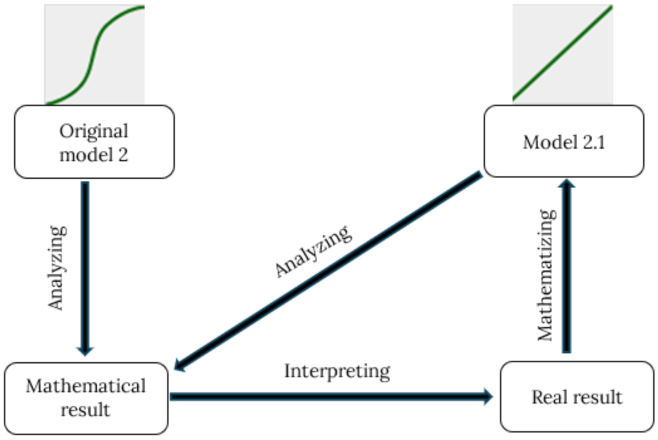


**Figure 3.** The visualized modeling cycle for environmental knowledge (Source: Authors' own elaboration based on the results of the present study)

When the relationship of this pattern to the types of knowledge defined under the first subproblem is considered, it is particularly associated with experiential knowledge. Within this atomic mathematical modeling process, which is linked to experiential knowledge, classroom discussions were more extended and more in depth. During these discussions, the students were observed to be more participatory, interpretive, creative, and enthusiastic.

### *The second pattern*

The second pattern refers to situations in which the real-life scenarios written for the given mathematical model (the graph) are compared with other graphs on the basis of the mathematical expressions contained in those scenarios (e.g., increase, decrease, steady, constant, or slow increase). In other words, this pattern primarily involves mathematical comparisons and reasoning (see **Figure 3**).

As illustrated in **Figure 3**, in this modeling pattern, instead of constructing original alternative models for the given scenario, the students selected another graph from among the 15 provided models, which they considered more suitable for the scenario. During this process, rather than developing new models, students engaged in intensive discussions about mathematical concepts. The primary mathematical concepts addressed in these discussions included "sudden and/or constant increase" (linear change), "gradual increase and/or stagnation" (nonlinear change), "total difference," and statements such as "… if we divide it into two, the upper and lower parts would increase regularly" (mathematical analysis of the graph). Similar to the first pattern, no sequential or cyclical structure was observed in this modeling process.

The students' dialogs were as follows:

> S4 (original model 2): "Here, the country is newly established; at first, its economy is at zero, then it gradually rises. After reaching this point, it continues to increase at a constant rate and then rises even more."

> S5 (model 2.1): "… may I ask a question? For example, if we presented 15 (original model 2) instead of 1 (model 2.1), what would be the difference …?"

When the students' dialogs were examined within the framework of the stages and competencies of the mathematical modeling cycle, a total of 23 behaviors were identified. Of these, 18 corresponded to analysis, 4 to interpretation, and 1 to mathematizing. Selected quotations representing each competency are provided below:

> S17: "… Teacher, may I ask a question? For example, if we presented 15 (original model 2) instead of 1 (model 2.1), what would be the difference?" (mathematizing)

> S21: "… Teacher, may I say something? In 1, it has constantly increased steadily, but in 15, after a certain point, it went upward, meaning it did not remain constant …" (analyzing)

> S21: "… the amount of money increased, but the economy did not increase …" (interpreting)

Although not as intense as in the first pattern, it was observed that the students used and discussed several mathematical concepts. For example, concepts such as "sudden and/or constant increase" (linear change), "gradual increase and/or stagnation" (nonlinear change), "dividing into two for regular increase in the upper and lower parts" (graphical analysis), and "total difference" were expressed. Furthermore, although to a lesser extent than in the first pattern, students also made comments about nonlinear graphs that were cognitively demanding for their level, using terms such as "gradual increase" or "stagnation."

When the relationship of this pattern with the knowledge types identified in the first subproblem was examined, it was found to be particularly associated with environmental knowledge. Within the atomic mathematical modeling activity process linked to environmental knowledge, students were observed to be less participatory, interpretive, creative, and motivated than they were in the processes associated with experiential knowledge.

### *The third pattern*

The third pattern refers to the type of modeling process that concludes with an explanation of a real-life scenario constructed for the given mathematical model (the graph) (see **Figure 4**). As shown in **Figure 4**, in this modeling pattern, the discussion process concludes with a few questions directed toward the scenario. Within the context of the science and technology course, students' responses were found to be quite limited and based on static mathematical knowledge. The student's presentation of the model was expressed as follows:

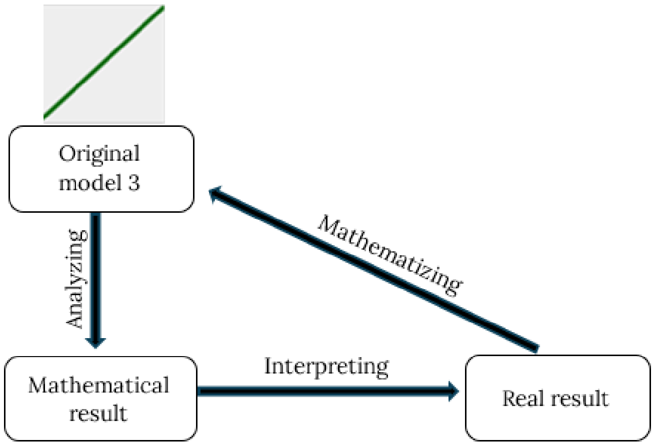


**Figure 4.** The visualized modeling cycle for academic knowledge (Source: Authors' own elaboration based on the results of the present study)

> S7 (original model 3). "It may be the graph of a vehicle moving at a constant speed …"

When the students' dialogs in this pattern were examined according to the stages and competencies of the mathematical modeling cycle, they demonstrated two instances of "analyzing" and one instance of "interpreting." Selected examples of student expressions corresponding to each competency are provided below:

> S18: "… We divide the graph into 1, 2, 3. One side represents the distance traveled, and the other represents time." (analyzing)

> S3: "… It is not going straight (the phrase 'straight' was interpreted as the absence of any condition that would change the vehicle's speed) …" (interpreting)

In this type of pattern, only a minimal number of mathematical concepts are used and discussed. For example, the students provided only simple explanations regarding the names and scales of the coordinate axes in the graph. The students operated within their cognitive level and employed previously learned concepts. When this pattern type is examined in relation to the knowledge categories defined in the first subproblem, it is particularly associated with Academic Knowledge. In the atomic mathematical modeling process linked to academic knowledge, students were observed to be considerably less participatory, interpretive, creative, and motivated than they were in scenarios associated with experiential or environmental knowledge.

## RESULTS AND DISCUSSION

This study investigated students' mathematical modeling skills in the context of constructing real-life situations related to a given mathematical model through an atomic approach. The findings are summarized as follows:

The first result indicates that sixth-grade students, when relating a given mathematical model to real-life situations, mostly draw upon experiential knowledge gained directly from their own experiences or environmental knowledge obtained from sources such as social media and television. In contrast, academic knowledge, which comprises theoretical knowledge learned in school subjects, played a more limited role in this process. This finding is consistent with English and Watters (2005), who argued that the modeling process is closely linked to students' personal knowledge and experiences, and with Garfunkel and Montgomery's (2016), who concluded that students often rely on personal experiences in their problem-solving approaches. Furthermore, as grade levels and educational stages increase, the amount and depth of theoretical knowledge acquired in school also increase. Accordingly, future research is recommended to examine whether the utilization of academic knowledge in linking mathematical models to real-life situations differs across grade levels or educational stages.

With respect to the second subproblem, three fundamental patterns were identified in the atomic analysis of students' processes of constructing real-life situations for a given mathematical model. An examination of these patterns revealed four key results.

First, in classroom discussions of real-life scenarios associated with experiential knowledge and environmental knowledge, students exhibited a greater number of behaviors corresponding to the stages and competencies of the mathematical modeling cycle. The number of competency-related behaviors was greater in scenarios based on experiential knowledge than in those based on environmental knowledge. Moreover, in discussions grounded in experiential knowledge, the competency of *interpretation* was more prominent, whereas in those related to environmental knowledge, *analysis* predominated. In both types of scenarios, students made efforts to compare and connect mathematics with real-life situations. Thus, compared with Environmental Knowledge, scenarios associated with Experiential Knowledge enabled students to demonstrate more stages and competencies of the mathematical modeling cycle, advancing as far as the highest-level competency, *interpretation*. In line with this result, it is suggested that the atomic approach be employed in overcoming the challenges identified by Blum et al. (2007) regarding the teaching of competencies in the mathematical modeling process and that modeling activities centering on students' own experiences be integrated into instruction.

In contrast, discussions based on academic knowledge demonstrated far fewer behaviors corresponding to the stages and competencies of the mathematical modeling cycle than those based on experiential or environmental knowledge. These behaviors were generally limited to examples encountered in class. This outcome aligns with the findings of the first subproblem. Thus, students demonstrate the highest levels of discussion, performance, and higher-order thinking skills in activities that draw on experiential knowledge, particularly in scenario writing and classroom discussions. From this perspective, when designing mathematical modeling activities, teachers are advised to avoid artificial and academically dominated contexts and instead design and implement activities grounded in students' environments and experiences.

Second, in discussions of scenarios related to real-life situations, deviations from sequence and cyclicity were observed among the competencies of "mathematical result," "real result," and "real situation" within the atomic mathematical modeling cycle. Irregular transitions occurred, and some competencies did not emerge at all. This suggests that competencies appear in different forms and sequences in different modeling activities. This result supports Galbraith et al.'s (2007) finding that focusing on every stage of the modeling cycle is not always necessary in classroom practice and research. Therefore, teachers are encouraged to adopt more flexible and adaptive approaches in implementing modeling activities rather than expecting students to complete all modeling stages sequentially and comprehensively.

Third, in atomic mathematical modeling activities, students were able to meaningfully and functionally employ both mathematical concepts they had already learned and those they had not yet formally encountered, without necessarily using the technical terminology. For example, even though they had not received formal instruction online graphs, students were able to make meaningful readings, interpretations, and inferences from such graphs. Indeed, one student suggested a piecewise function graph—typically introduced in the 10th-grade curriculum—as an alternative mathematical model for a given scenario. Furthermore, the students used and interpreted mathematical concepts such as "decrease," "increase," "constant," "the graph not touching the x- or y-axis," "the highest price being represented at the maximum point of the graph and/or the lowest at the minimum point," "sudden or constant increase in a linear graph," "gradual increase or stagnation in a nonlinear curve," and "difference of totals" in a meaningful way. These findings are consistent with prior research (Carpenter & Romberg, 2004; English & Watters, 2005; Manouchehri et al., 2020; Wickstrom, 2017), which showed that elementary and middle school students can make sense of, apply, and integrate mathematical concepts into their existing knowledge and experiences through modeling. This study demonstrates that the atomic mathematical modeling process not only enables sixth-grade students to use their existing knowledge and experiences but also supports their use and learning of higher-level mathematical concepts, rules, relationships, and generalizations. Accordingly, it is suggested that teachers at the middle school level implement modeling activities on the basis of the atomic approach in the teaching of new mathematical topics.

Fourth, during large-group discussions in the atomic mathematical modeling process, students first evaluate the scenario written for the given mathematical model and then propose existing or newly constructed graphical models related to the scenario.

The first noteworthy point here is that although the atomic approach initially required students to write a real-life scenario for a given mathematical model, students proceeded immediately to propose alternative models, thereby engaging in a holistic modeling cycle. In other words, a process that began atomically was transformed into a holistic modeling process. This can be seen, for example, in one student's response to the teacher's question, "*Couldn't this graph be read in this way?*" The student replied, "*This graph could be read in that way, but it would not be convincing.*" When unconvinced by the explanation, the student returned to the beginning of the process to search for alternative solutions, thus repeating a cyclical process. This result aligns with Blum and Leiß's (2007) observation that students can reconstruct the modeling process by revisiting earlier steps when needed. Accordingly, teachers should encourage students to develop alternative models and organize group activities in which such models can be discussed and compared. In this way, students can transition from atomic modeling to a holistic modeling cycle, thereby enhancing their mathematical thinking skills more effectively.

The second noteworthy point is that most of the scenarios in which students developed alternative models were associated with experiential knowledge and environmental knowledge. This finding supports Garfunkel and Montgomery's (2016) and Carlisle et al.'s (2025) conclusions that modeling situations should be designed in ways that connect to students' own experiences.

The third noteworthy point is that students tend to accept mathematical concepts, relationships, and expressions learned theoretically in school without critical examination. For example, in the third pattern, the linear graph model was widely accepted by students as mathematically correct, even though maintaining constant speed is rarely possible in real life. Because this concept was formally learned in the science and technology course, it was accepted without debate or interpretation. To counter this tendency, teachers are encouraged to create opportunities for critical thinking and discussion within modeling activities, thereby fostering students' ability to reflect their experiences in mathematical models and to evaluate the real-life validity of mathematical concepts.

## LIMITATIONS AND FUTURE RESEARCH

This study, which is based on an atomic approach to the topic of graphs, was conducted with sixth-grade students in a single province and employed a cross-sectional design. This constitutes a limitation in terms of the generalizability of the findings. Therefore, similar studies should be conducted in different geographical regions, at various grade levels, and use a longitudinal design to increase the generalizability of the results through comparative analysis. Furthermore, in this study, where students engaged in group work and peer learning, teacher interventions were limited. For future research on different mathematical topics using the atomic approach, investigating implementations with more explicit teacher interventions (Czocher et al., 2025; Manouchehri et al., 2020) and conducting comparative studies are recommended

**Author contributions:** Both authors contributed equally to all stages of the study, including conceptualization, methodology, data collection, analysis, writing, and revision. Both authors approved the final version of the article.

**Funding:** The authors received no financial support for the research and/or authorship of this article.

**Ethics declaration:** The authors state that this study was conducted in accordance with the academic integrity and ethical research principles of Erzincan Binali Yıldırım University. Participation was voluntary, and informed consent was obtained from all participants prior to data collection. All data were anonymized before analysis, and no personally identifiable information was collected, stored, or reported. Participant confidentiality and privacy were maintained throughout the study.

**AI statement:** The authors used a generative AI tool solely for language editing and improvement of grammar, spelling, and readability. The AI tool was not used for the generation of research ideas, study design, data collection, data analysis, interpretation of results, or manuscript conclusions. All scientific content, analyses, interpretations, and final decisions regarding the manuscript were developed and verified by the authors, who take full responsibility for the content of the article.

**Declaration of interest:** The authors declare no competing interest.

**Data availability:** Data generated or analyzed during this study are available from the authors on request.

## REFERENCES

Albarracín, L., & Gorgorió, N. (2019). Using large number estimation problems in primary education classrooms to introduce mathematical modeling. *International Journal of Innovation in Science and Mathematics Education, 27*(2), 45-57. https://doi.org/10.30722/IJISME.27.02.004

Anhalt, C. O., & Cortez, R. (2016). Developing understanding of mathematical modeling in secondary teacher preparation. *Journal of Mathematics Teacher Education, 19*, 523-545. https://doi.org/10.1007/s10857-015-9309-8

Bekdemir, M., Sağlam Kaya, Y., & Manouchehri, A. (2024). Developing a scale to determine the knowledge of the preservice teachers related to mathematical modelling. *Hacettepe University Journal of Education*, 39(4), 353-365. https://doi.org/10.16986/huje.2024.529

Garfunkel, S., & Montgomery, M. (Eds.) (2016). *GAIMME: Guidelines for assessment & instruction in mathematical modeling education*. Consortium for Mathematics and Its Applications & Society for Industrial and Applied Mathematics. https://www.siam.org/media/n2tinywa/gaimme-2nd-ed-final-download-print-bw.pdf

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modeling competence: Conceptual clarification and educational planning. *Teaching Mathematics and its Applications*, 22(3), 123-139. https://doi.org/10.1093/teamat/22.3.123

Blomhøj, M., & Kjeldsen, T. H. (2007). *Learning the integral concept through mathematical modelling*. In CERME 5–Proceedings of the fourth congress of the European Society for Research in Mathematics Education (pp. 2070-2079). https://erme.site/wp-content/uploads/CERME5/WG13.pdf

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modeling: Can it be taught and learnt? *Journal of Mathematical Modeling and Application*, 1(1), 45-58. https://eclass.uoa.gr/modules/document/file.php/MATH601/3rd%20&%204rth%20unit/3rd%20unit_Modelling%20cycle.pdf

Blum, W., & Leiß, D. (2007). How do students' and teachers deal with modeling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling: Education, engineering and economics* (pp. 222-231). Horwood Publishing. https://doi.org/10.1533/9780857099419.5.221

Blum, W., Galbraith, P. L., Henn, H.-W., & Niss, M. (2007). *Modeling and applications in mathematics education*. Springer. https://doi.org/10.1007/978-0-387-29822-1

Cakmak Gurel, Z., & Bekdemir, M. (2022). The teacher and peer intervention for pre-service mathematics teachers on the validity of mathematical models. *Pedagogical Research*, 7(2), Article em0120. https://doi.org/10.29333/pr/11800

Carlisle, W. R., Jung, H., Wickstrom, M. H., Sutcliffe, K., & Kim, H. J. (2025). Insights into pothole damage: Exploring culturally responsive mathematical modeling. *International Journal of Science and Mathematics Education*, 23(4), 1013-1032. https://doi.org/10.1007/s10763-024-10492-0

Carlson, M. A., Wickstrom, M. H., Burroughs, E. A., & Fulton, E. W. (2016). A case for mathematical modeling in the elementary school classroom. In C. R. Hirsch and A. R. McDuffie (Eds.), *Annual Perspectives in mathematics education 2016: Mathematical modeling* (pp. 121-129). National Council of Teachers of Mathematics. https://www.nctm.org/Store/Products/Annual-Perspectives-in-Mathematics-Education-2016--Mathematical-Modeling/

Carpenter, T. P., & Romberg, T. A. (2004). *Powerful practices in mathematics and science: Research-based practice for teaching and learning*. Learning Point https://www.amazon.com/Powerful-Practices-Mathematics-Science-Book/dp/B003UV1IPM

Common Core State Standards Initiative (2010). *Common Core State Standards for Mathematics*. National Governors Association Center for Best Practices and Council of Chief State School Officers. https://corestandards.org/mathematics-standards/

Cirillo, M., Pelesko, J. A., Felton-Koestler, M. D., & Rubel, L. (2016). Perspectives on modeling in school mathematics. In C. R. Hirsch, & A. R. McDuffie (Eds.), *Annual perspectives in mathematics education 2016: Mathematical modeling* (pp. 3-16). National Council of Teachers of Mathematics. https://www.nctm.org/Handlers/AttachmentHandler.ashx?attachmentID=fePJMG5zJqM%3D

Czocher, J. A., White, A., Kularajan, S. S., Roan, E., & Baas, A. (2025). Quantizing and visualizing the influence of scaffolding moves on mathematical modeling competencies. *Methods in Psychology*, 13, Article 100210. https://doi.org/10.1016/j.metip.2025.100210

Durandt, R., & Lautenbach, G. (2020). Strategic support to students' competency development in the mathematical modeling process: A qualitative study. *Perspectives in Education*, 38(1), 211-223. https://doi.org/10.18820/2519593X/pie.v38i1.15

English, L. D. (2012). Data modeling with first-grade students. *Educational Studies in Mathematics*, 81(1), 15-30. https://doi.org/10.1007/s10649-011-9377-3

English, L. D. (2021). Mathematical and interdisciplinary modeling in optimizing young children's learning. In J. M. Suh, M. H. Wickstrom, & L. D. English (Eds.), *Exploring mathematical modeling with young learners, early mathematics learning and development* (pp. 3-23). Springer. https://doi.org/10.1007/978-3-030-63900-6_1

English, L. D., & Doerr, H. M. (2003, July 13-18). Perspective-taking in middle school mathematical modeling: A teacher case study [Conference presentation]. *Proceedings of the 27th International Group for the Psychology of Mathematics Education Conference*, 2, 357-364. https://eric.ed.gov/?id=ED500952

English, L. D., & Watters, J. J. (2005). Mathematical modeling in the early school years. *Mathematics Education Research Journal*, 16(3), 58-79. https://doi.org/10.1007/BF03217401

English, L. D., Ärlebäck, J. B., & Mousoulides, N. (2016). Reflections on progress in mathematical modeling research. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education: The journey continues* (pp. 383-413). Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_11

Erbaş, A. K., Kertil, M., Çetinkaya, B., Çakiroglu, E., Alacaci, C., & Bas, S. (2014). Mathematical modeling in mathematics education: Basic concepts and approaches. *Educational Sciences: Theory and Practice*, 14(4), 1621-1627. https://eric.ed.gov/?id=EJ1045031

Galbraith, P., Henn. H.-W., & Niss, M. (2007). *Modeling and applications in mathematics education*. Springer. https://doi.org/10.1007/978-0-387-29822-1

Geiger, V., Bergman, J., & Frejd, P. (2016). Interpreting curricula to find opportunities for modeling: Case studies from Australia and Sweden. In C. R. Hirsch & A. R. McDuffie (Eds.), *Annual perspectives in mathematics education 2016: Mathematical modeling* (pp. 207-216). National Council of Teachers of Mathematics. https://www.nctm.org/Store/Products/Annual-Perspectives-in-Mathematics-Education-2016--Mathematical-Modeling/

Gould, H. T. (2013). *Teachers' conceptions of mathematical modeling* [PhD thesis, Columbia University]. https://academiccommons.columbia.edu/doi/10.7916/D87D32BS

Greefrath, G., & Vorhölter, K. (2016). Teaching and learning mathematical modeling: Approaches and developments from German speaking countries. In G. Greefrath & K. Vorhölter (Eds.), *Teaching and learning mathematical modelling. ICME-13 Topical Surveys* (pp. 1-42). Springer. https://doi.org/10.1007/978-3-319-45004-9_1

Haines, C., Crouch, R., & Fitzharris, A. (2003). Deconstructing mathematical modelling: Approaches to problem solving. In Q.-X. Ye, W. Blum, K. Houston, & Q.-Y. Jiang (Eds.), *Mathematical modelling in education and culture* (pp. 41–53). Woodhead Publishing. https://doi.org/10.1533/9780857099556.1.41

Kaiser, G. (2017). The teaching and learning of mathematical modeling. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 267-291). National Council of Teachers of Mathematics. https://www.nctm.org/Store/Products/Compendium-for-Research-in-Mathematics-Education/

Leavy, A., & Hourigan, M. (2018). The role of perceptual similarity, context, and situation when selecting attributes: Considerations made by 5-6-year-olds in data modeling environments. *Educational Studies in Mathematics*, 97(2), 163-183. https://doi.org/10.1007/s10649-017-9791-2

Lehrer, R., & Schauble, L. (2007). Scientific thinking and scientific literacy: Supporting development in learning in context. In W. Damon, R. M. Lerner, K. A. Renninger, & I. E. Sigel (Eds.), *Handbook of child psychology* (pp. 153-196). John Wiley and Sons. https://doi.org/10.1002/9780470147658.chpsy0405

Leiß, D., & Wiegand, B., (2005). A classification of teacher interventions in mathematics teaching. *ZDM Mathematics Education*, 37, 240-245. https://doi.org/10.1007/s11858-005-0015-3

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Lawrence Erlbaum. https://doi.org/10.4324/9781410607713

Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical Thinking and Learning*, 5(2-3), 157-189. https://doi.org/10.1080/10986065.2003.9679998

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical Thinking and Learning*, 5(2-3), 109-129. https://doi.org/10.1080/10986065.2003.9679996

Lesh, R., & Yoon, C. (2007). What is distinctive in (our views about) models & modeling perspectives on mathematics problem solving, learning, and teaching? In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education* (pp. 161-170). Springer. https://doi.org/10.1007/978-0-387-29822-1_15

Lingefjärd, T. (2007). Mathematical modeling in teacher education—Necessity or unnecessarily. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education* (pp. 333-340). Springer. https://doi.org/10.1007/978-0-387-29822-1_35

Manouchehri, A. (2017). Implementing Mathematical Modeling: The challenge of teacher educating. In G. A. Stillman (Ed.), *Mathematical modeling and applications: Crossing and Researching Boundaries in Mathematics Education* (pp. 421-432). Springer. https://www.researchgate.net/publication/320899350_Implementing_Mathematical_Modelling_The_Challenge_of_Teacher_Educating

Manouchehri, A., Bekdemir, M., & Yao, X. (2020). Facilitating modeling activities in a grade 5 classroom. In G. A. Stillman, G. K. Christine & E. Lampen (Eds.), *Mathematical modeling education and sense-making* (pp. 187-197). Springer. https://link.springer.com/chapter/10.1007/978-3-030-37673-4_17

Merriam, S. B., & Grenier, R. S. (2019). *Qualitative research in practice: Examples for discussion and analysis*. Jossey-Bass Publishers. https://www.wiley.com/en-no/shop/general-introductory-education/qualitative-research-in-practice-examples-for-discussion-and-analysis-2nd-edition-p-9781119452027

Mousoulides, N. G., Christou, C., & Sriraman, B. (2008). A modeling perspective on the teaching and learning of mathematical problem solving. *Mathematical Thinking and Learning*, 10(3), 293-304. https://doi.org/10.1080/10986060802218132

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 3-32). Springer. https://doi.org/10.1007/978-0-387-29822-1_1

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. Sage Publications. https://study.sagepub.com/patton4e

Sokolowski, A. (2015). The effects of mathematical modeling on students' achievement-meta-analysis of research. *IAFOR Journal of Education*, 3(1), 93-114. https://doi.org/10.22492/ije.3.1.06

Steen, L. A., Turner, R., & Burkhardt, H. (2007). Developing mathematical literacy. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI study* (pp. 285-294). Springer. https://doi.org/10.1007/978-0-387-29822-1_30

Stender, P., & Kaiser, G. (2015). Scaffolding in complex modeling situations. *ZDM Mathematics Education*, 47, 1255-1267. https://doi.org/10.1007/s11858-015-0741-0

Stillman, G. (2000). Impact of prior knowledge of task context on approaches to applications tasks. *The Journal of Mathematical Behavior*, 19(3), 333-361. https://doi.org/10.1016/S0732-3123(00)00049-3

Suh, J. M., Matson, K., & Seshaiyer, P. (2017). Engaging elementary students in the creative process of mathematizing their world through mathematical modeling. *Education Sciences*, 7(2), Article 62. https://doi.org/10.3390/educsci7020062

Wickstrom, M. H. (2017, October 5-8). *Mathematical modeling: Challenging the figured worlds of elementary mathematics* [Conference session]. Paper presented at the Annual Meeting of the North American *chapter* of the international group for the psychology of mathematics education (pp. 685-692), Indianapolis, IN, United States. https://eric.ed.gov/?id=ED581352

Zawojewski, J. (2013). Problem solving versus modeling. In R. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (pp. 237-243). Springer. https://doi.org/10.1007/978-94-007-6271-8_20

## APPENDIX A. Graphs Presented to Students